# Modeling & Analysis of Wrinkled Membranes – An Overview

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2001 FEMCI Workshop

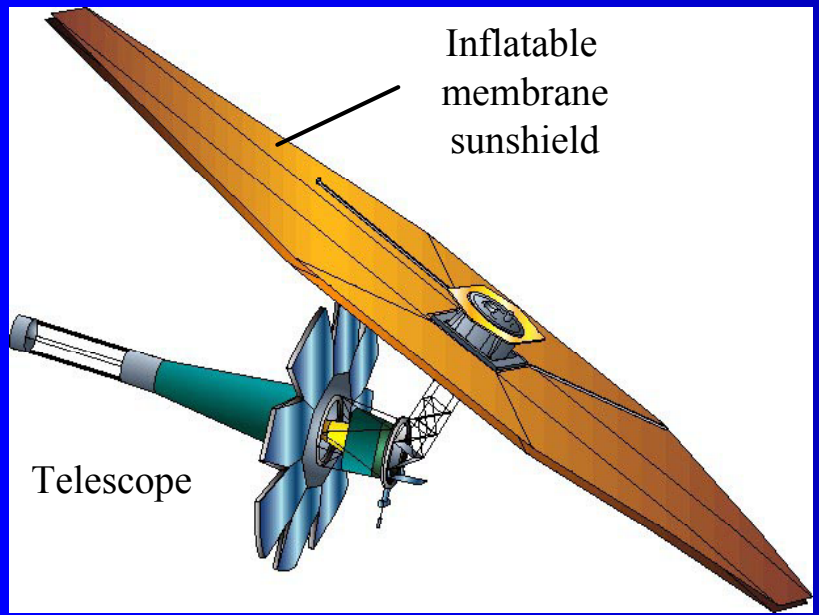
# Outline

- Background
- Literature Review

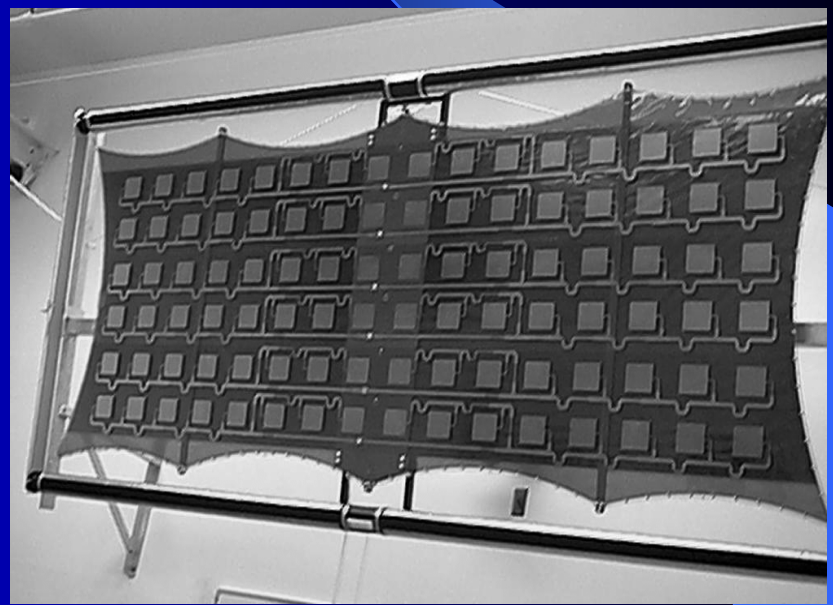
  --- Existing membrane models and membrane analysis tools
- Open Issues
- A New Modeling & Analysis Tool
- Conclusions

# Background

Membranes are a basic element of space inflatables and gossamer spacecraft

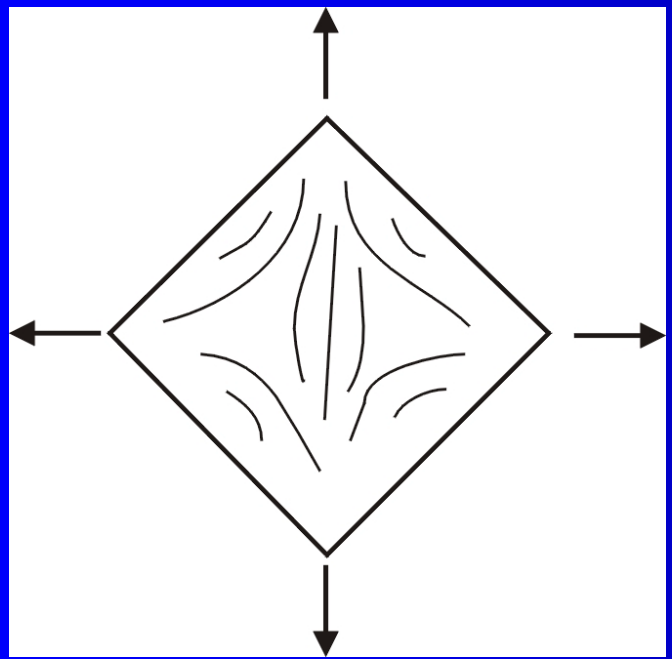



**The Next Generation Space Telescope (NGST)**



**The Inflatable Synthetic Aperture Radar (ISAR)**

# Wrinkling is a Critical Problem in Design of Membrane Structures



- Poor surface accuracy
- Degraded performance
- Significant effects on structural behaviors

# Motivation: Urgent Need for Wrinkling Analysis Tools

- Full-scale tests difficult
- Gravity & aerodynamic effects
- Optimal design

# Research Objectives

- Wrinkle formation and patterns
- Static and dynamic behaviors of wrinkled membranes
- Guidance for membrane design and wrinkling control

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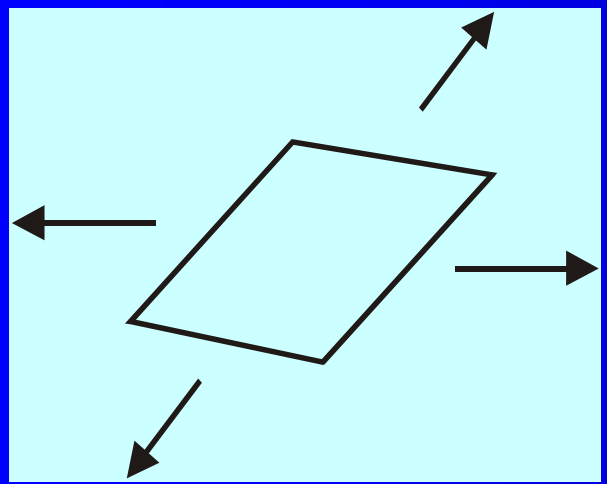
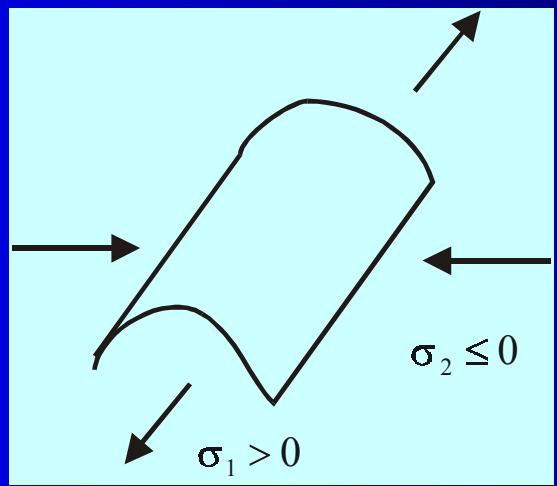
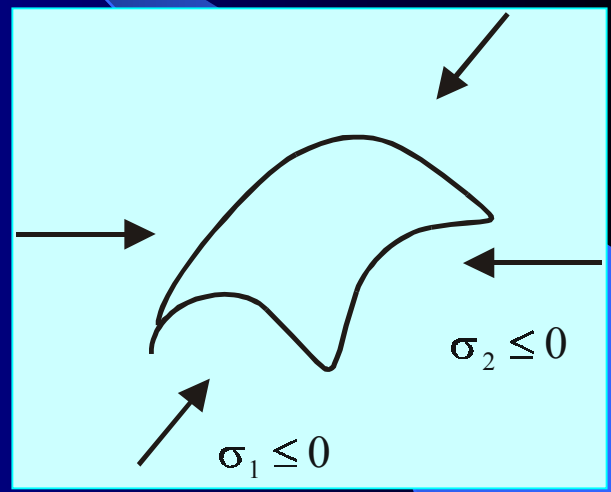
# Three States of Thin Membranes



**Taut State**



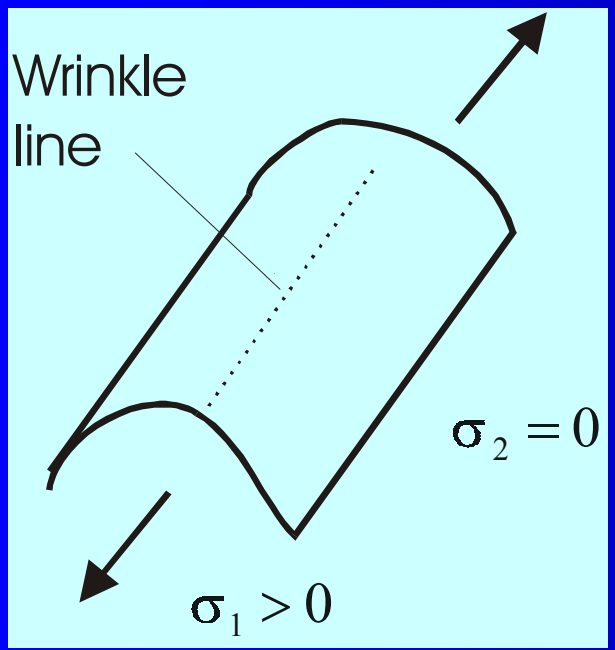

**Wrinkled State**




**Slack State**

# Tension Field Theory

(Wagner, 1929; Reissner, 1939)




- Wrinkling occurs when $\sigma_2 = 0$
- Wrinkle orientation follows $\sigma_1$ - lines

# Modified Elasticity Matrix

$$\{\sigma\} = [D]\{\varepsilon\} \Rightarrow \{\sigma\} = \left[\tilde{D}\right]\{\varepsilon\}$$

- **Stein-Hedgepeth Model (1961)**
  **Viable Poisson's ratio**
  Cannot model slack state

- **Miller & Hedgepeth (1985)**
  **Equivalent elasticity matrix**

$$\left[\tilde{D}\right] = [D_t],\ [D_w] \text{ or } [D_s]$$

  Selection of matrices not unique
  Material homogeneity destroyed
  Discontinuous among three states

# Relaxed Energy Density

(Pipkin, 1986; Steigmann and Pipkin 1989)

$$\{\sigma\} = [D]\{\varepsilon\} \Rightarrow \{\sigma\} = [D]\{\tilde{\varepsilon}\}$$

- No general algorithm available for numerical analyses

# Cable Network Model

(Alder, Mikulas and Hedgepeth, 2000)

- **Engineering model for dynamic analysis**

  **--- NGST sunshield (Johnston & Lienard, 2001)**

- **Prior knowledge of wrinkled patterns needed**

# Iterative Membrane Properties (IMP) Method

(Alder, Mikulas and Hedgepeth, 2000)

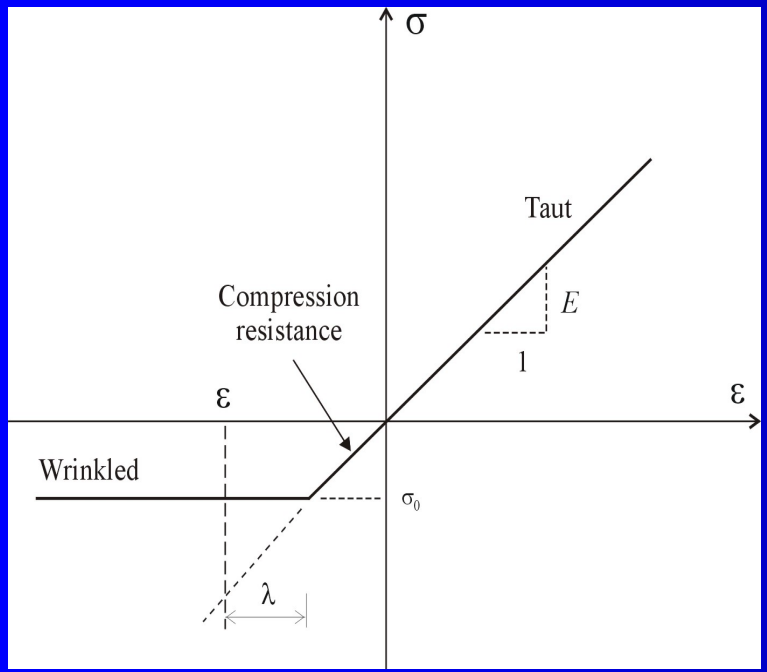
- **2D elastic elements in ABAQUS**
- **Slack state modeled by**

  Miller-Hedgepeth Model

  which could lead to numerical problems

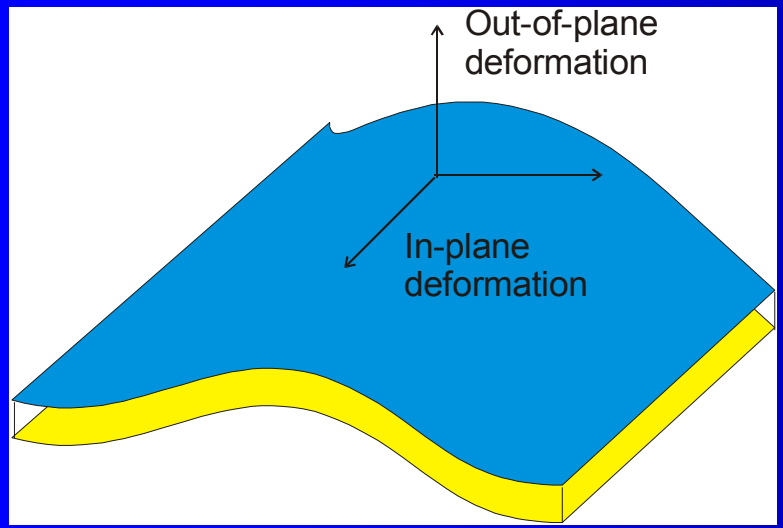
# Bar-Networking Approach

## (Yang, Ding, Lou & Fang, 2001)




- Viable control parameter λ
- Convergent solutions by math programming
- Wrinkle formation and patterns modeled

# Out-of-Plane Deformation




$$D\nabla^4 w + NL_b(u,v,w) = 0$$

$$NL_m(u,v,w) = 0$$

$$D << 1$$

- **Plate/shell elements**
- **Operator singularity occurs as *D* shrinks => numerical problems**
- **Many elements needed to model wrinkle waves**

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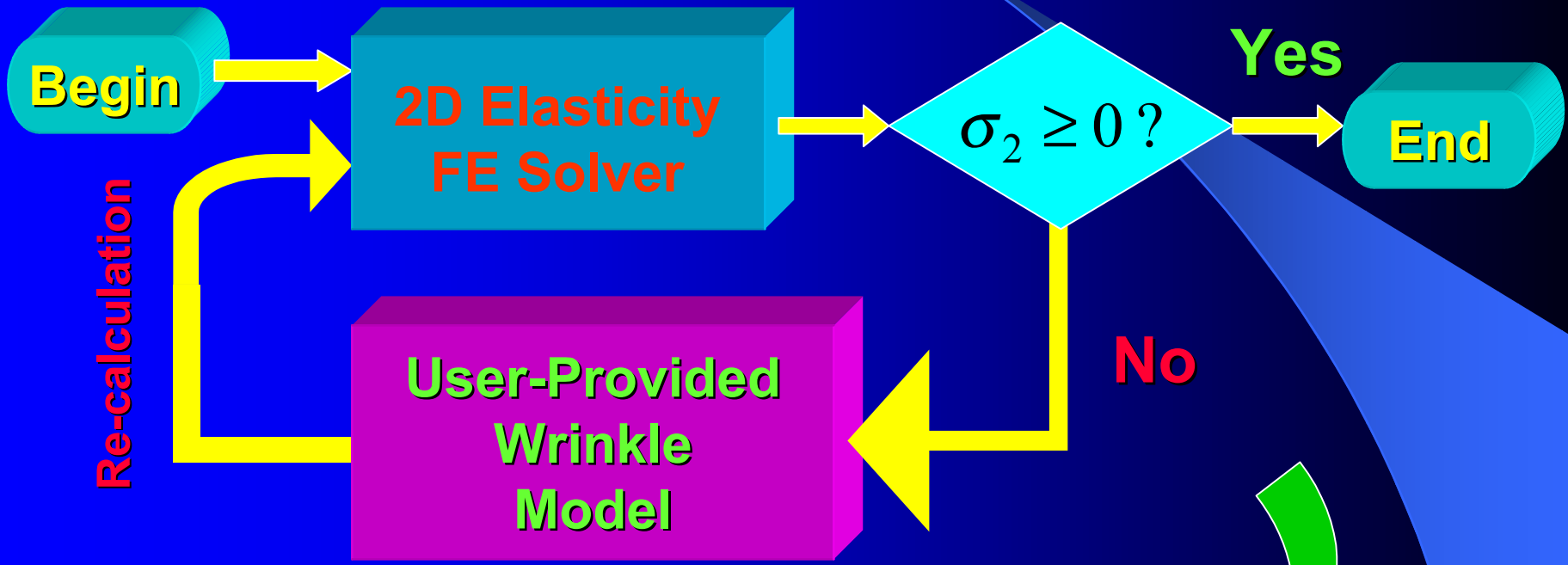
# Use of Finite Element Codes




Iteration

**Software-dependent**
**Convergence not sure**

# Problem with Modified Elasticity Matrix:
## Energy Non-minimum => Divergence

- **Conventional elasticity theory**

$$U = \{\varepsilon\}^T [D]\{\varepsilon\} > 0$$ pos. def.

- **After modification**

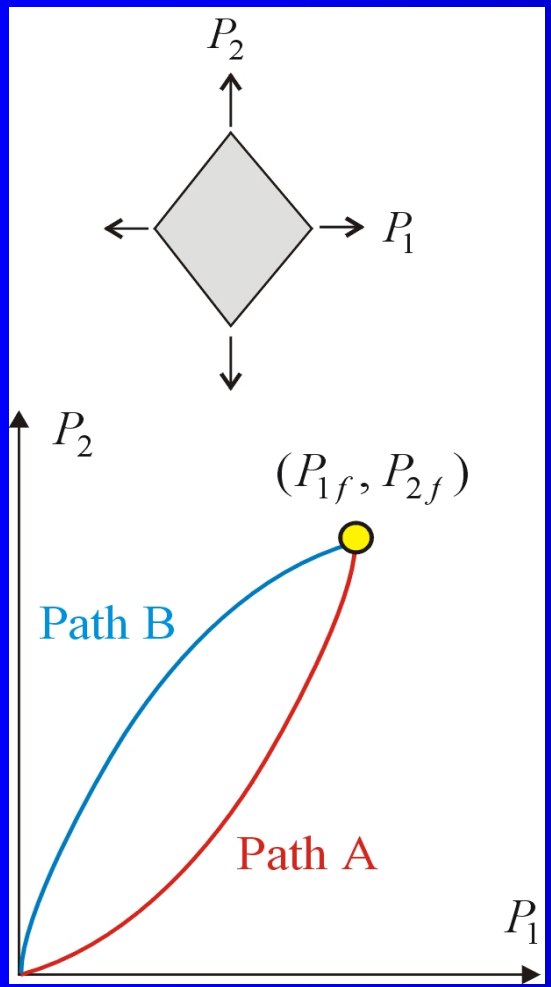
$$\tilde{U} = \{\varepsilon\}^T \left[\tilde{D}\right]\{\varepsilon\}$$ not surely pos. def.

**Convergence not guaranteed**

# Need to Model Slack State

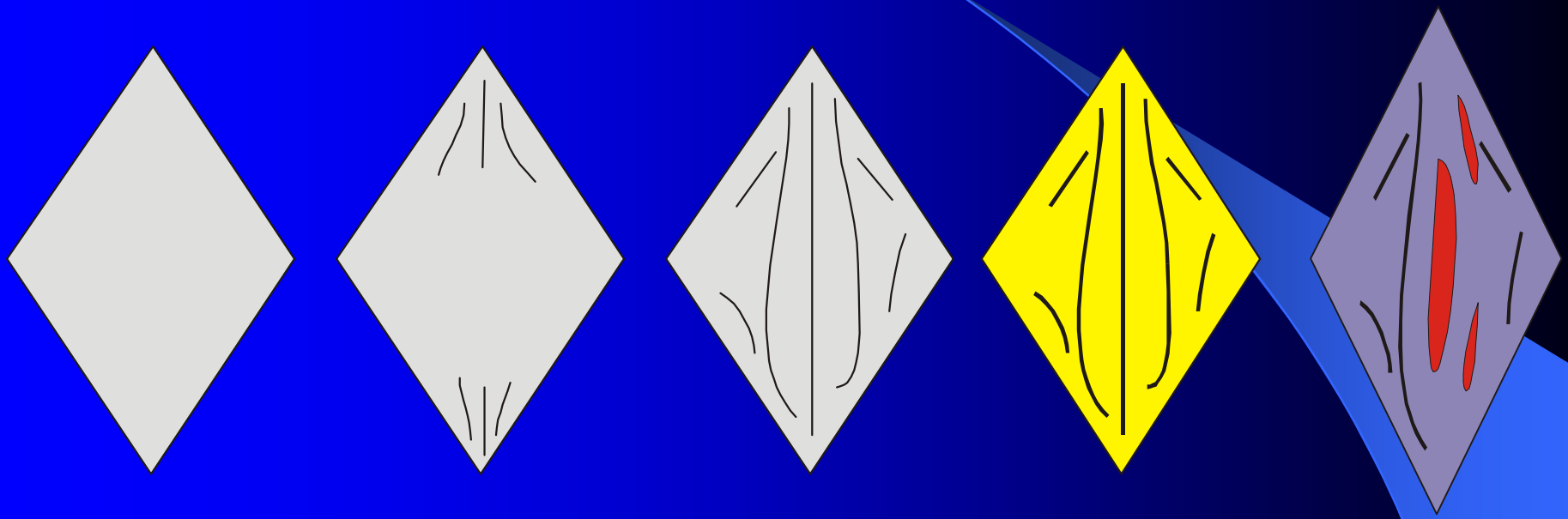
- Physically, many membrane structures have slack regions
- Numerically, slack state is a necessary transition in solution process

# Need to Know Wrinkle Formation




- Wrinkle patterns dependent upon loading path
- Optimal design
- Surface control by boundary loads

# Five Stages of Wrinkling Process



$S_0$
Undeformed

$S_W$
Wrinkled

$S_F$
Fully stretched

$S_P$
Post-stretching

$S_D$
Damaged

**GAP: No modeling & analysis tools covering wrinkling process from $S_w$ to $S_F$ to $S_P$**

# State of the Art

| | Cable Network Model | FEM with Modified Elasticity Matrix | Bar-Networking Approach | Desired Modeling & Analysis Tool |
|---|---|---|---|---|
| **Guaranteed Convergence** | Yes | Not sure | **Yes** | **Yes** |
| **Modeling of Slack State** | No | **Yes** # | Yes | **Yes** |
| **Wrinkle Formation** | No $S_P$ | Partially $S_W$ | Partially $S_W$, $S_F$ | **Fully** $S_W$, $S_F$, $S_P$ |
| **Wrinkle Waves** | No | No | No | **Yes** |
| **Dependent on FE Solver** | Yes | Yes | No | **No** |

# Miller-Hedgepeth Model could lead to numerical problems

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# New Modeling & Analysis Tool

Two-viable-parameter membrane model

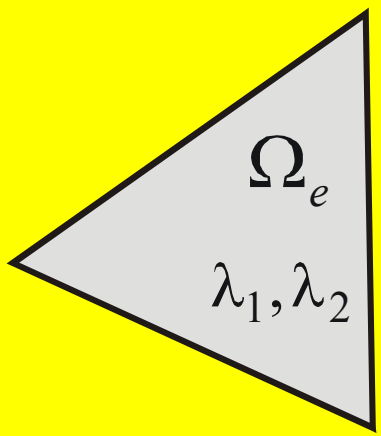
$$\{\sigma\} = [D(\lambda_1, \lambda_2)]\{\varepsilon\}$$

taut: $\lambda_1 = \lambda_2 = 0$

wrinkled: $\lambda_1 = 0, \lambda_2 > 0$

slack: $\lambda_1 > 0, \lambda_2 > 0$

Parametric FE formulation




Solution via nonlinear math programming

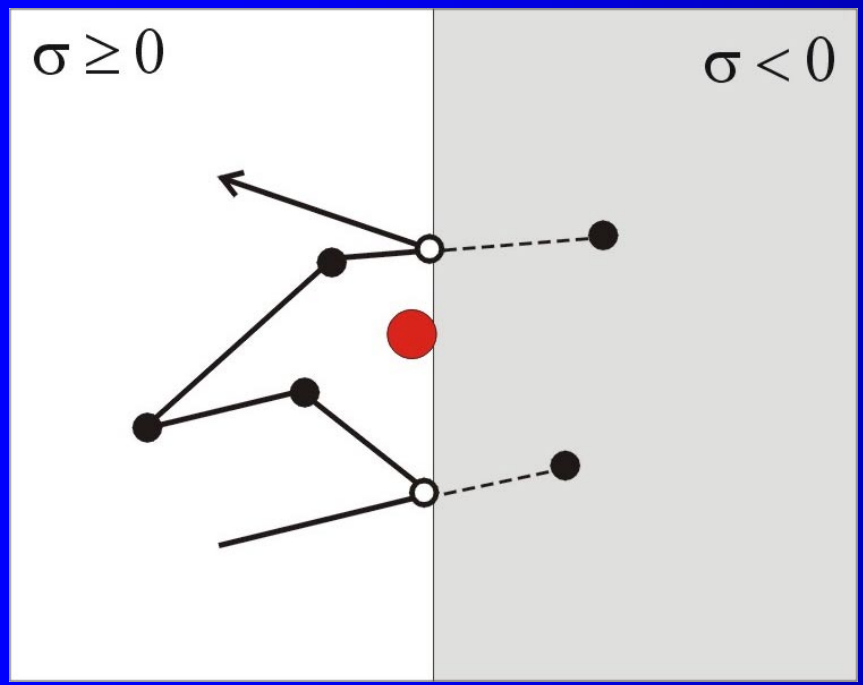
$$\Gamma(\{\lambda\}) \cdot \{\lambda\} = 0$$

**No iteration of membrane stresses**

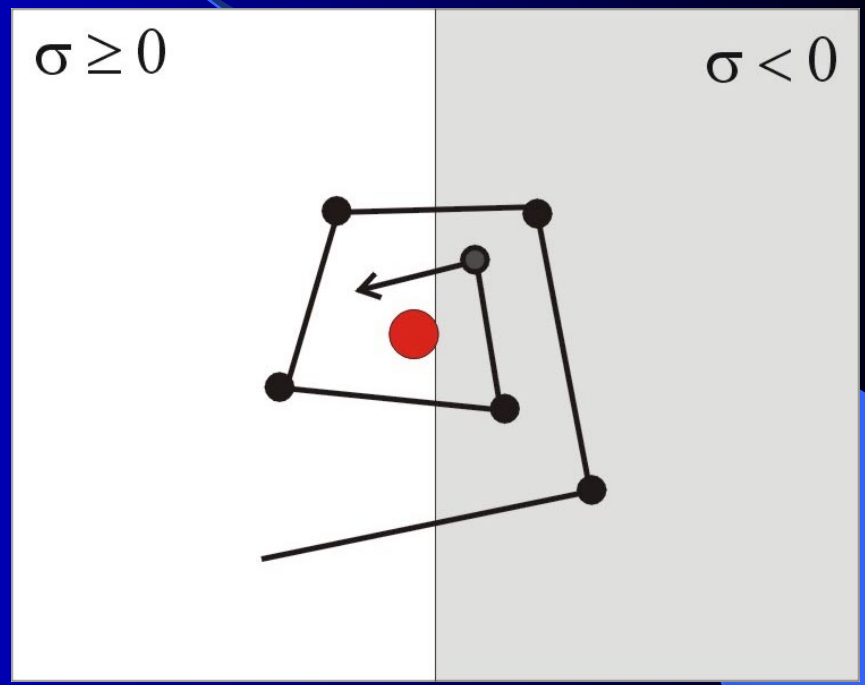
# Uniqueness and Capabilities

- Complete description of wrinkling process
- Convergent solution method
- Determination of out-of-plane deformation
- Modeling of multi-layered and/or composite membranes, thermal effects, curved membrane surfaces, ……

# Why Convergence Guaranteed




Iteration Based




New Tool

Wrinkle Solution

Numerical Solution

# New: Prediction of Out-of-Plane Deformation

**Shell/plate model**

$$D\nabla^4 w + NL_b(u,v,w) = 0$$
$$NL_m(u,v,w) = 0$$

viable parameters

**New membrane model**

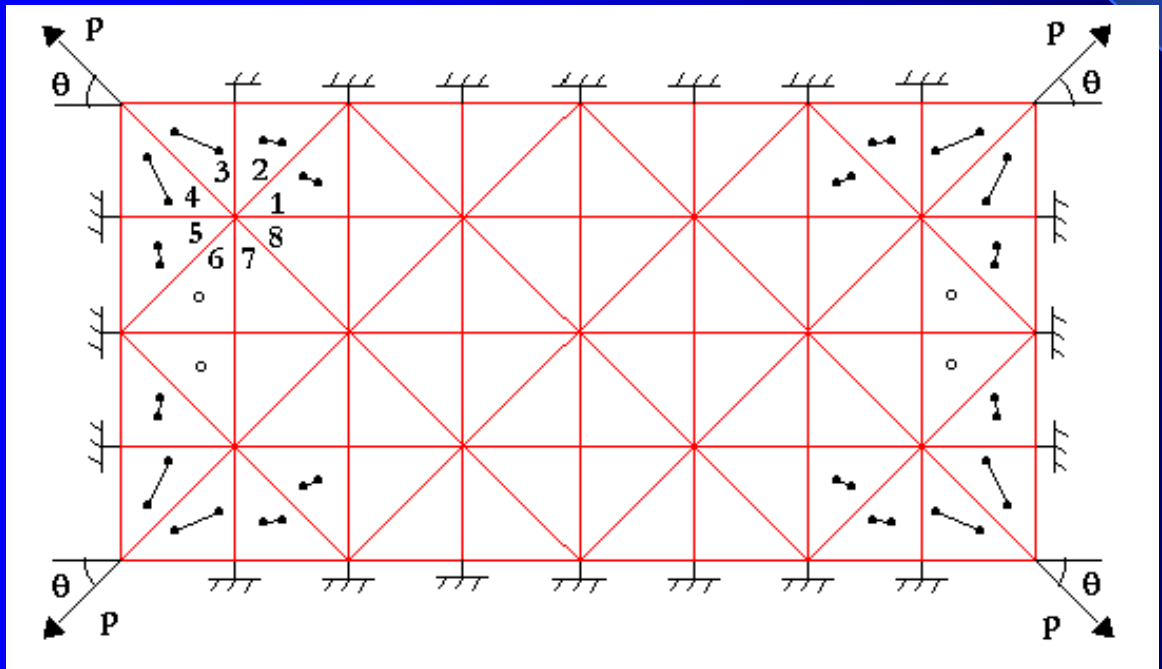
$$\left([K_m(\lambda)] + [K_b(\lambda)]\right)\{U\} = \{P\}$$

**Nonlinear optimization problem**

$$\Gamma(\{\lambda\}) \cdot \{\lambda\} = 0$$

→ **Solution**

# Example 1: Existence of slack elements

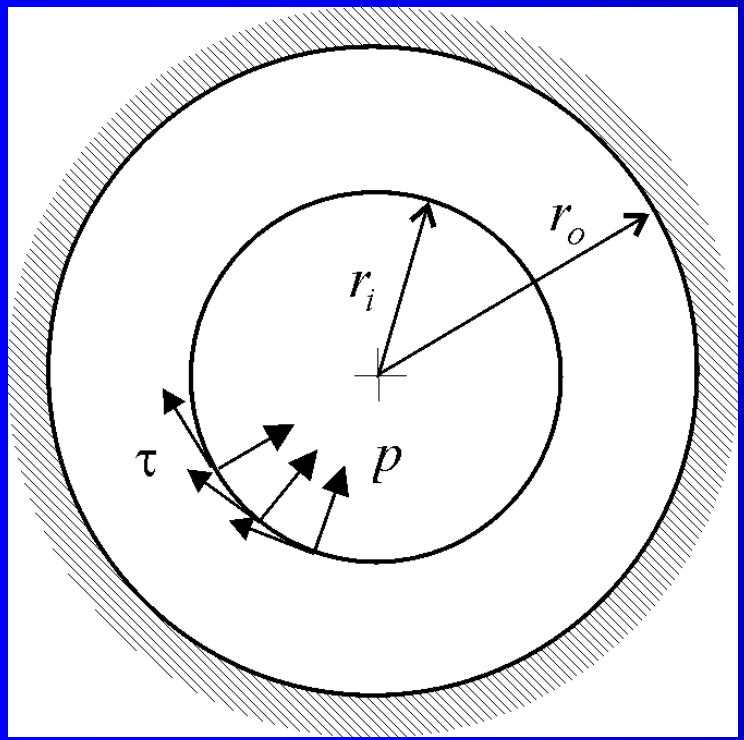


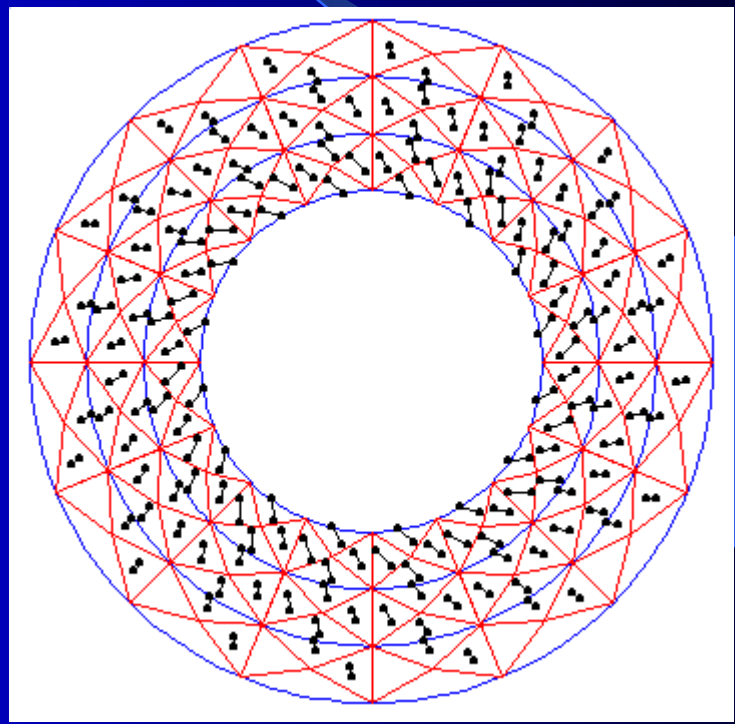

Taut: blank

Wrinkled:

Slack:

# Example 2: Wrinkle Evolution

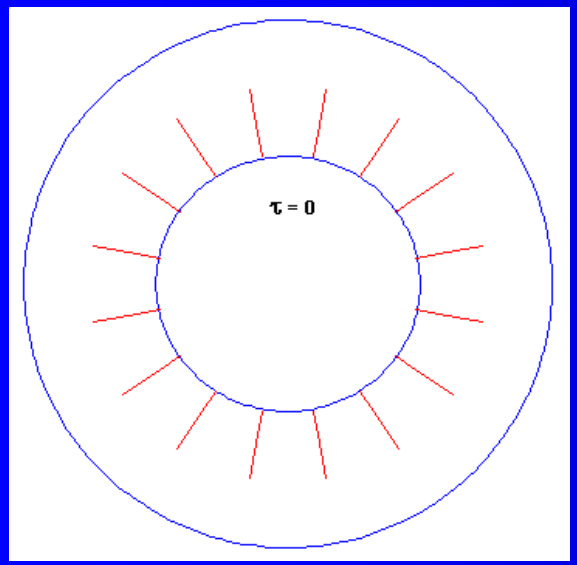


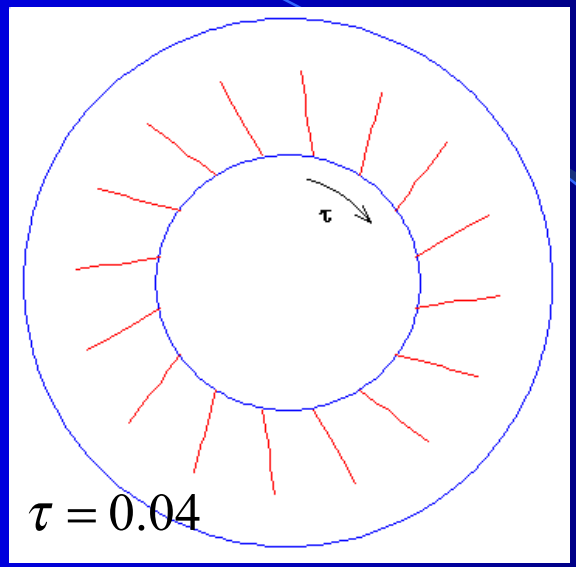



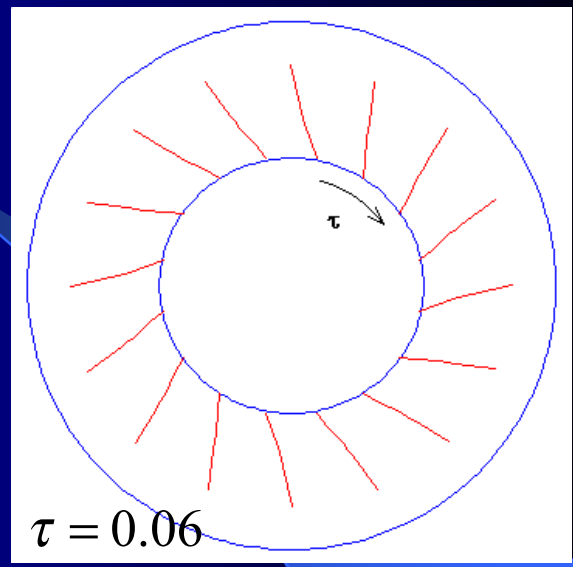
$p$ fixed at 0.01; $\tau$ varied from 0

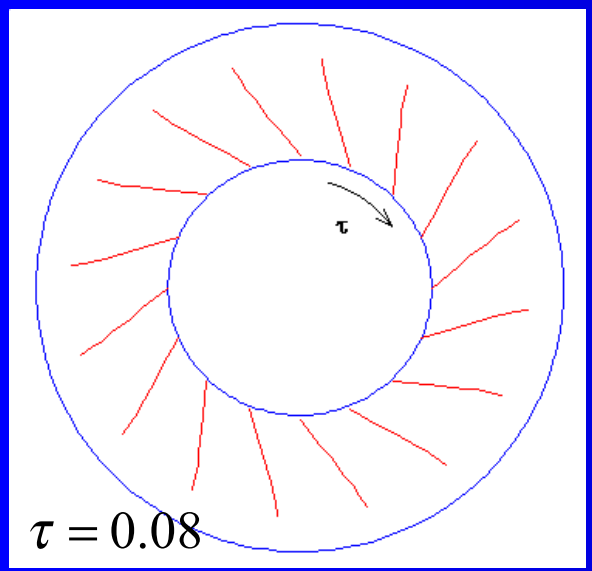
Example 2

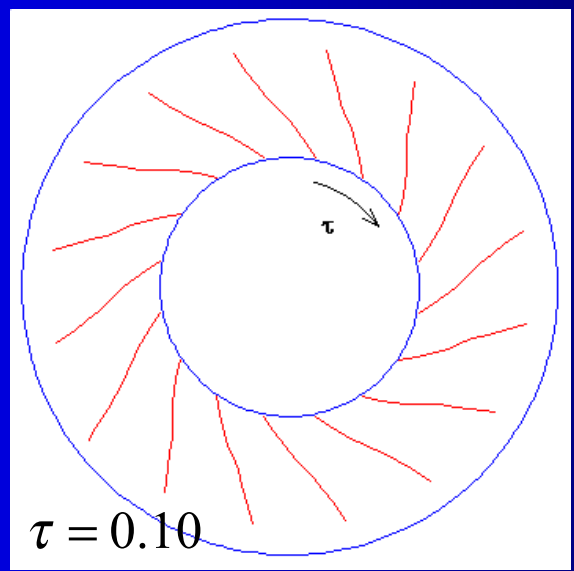
# Evolution of Wrinkle Lines

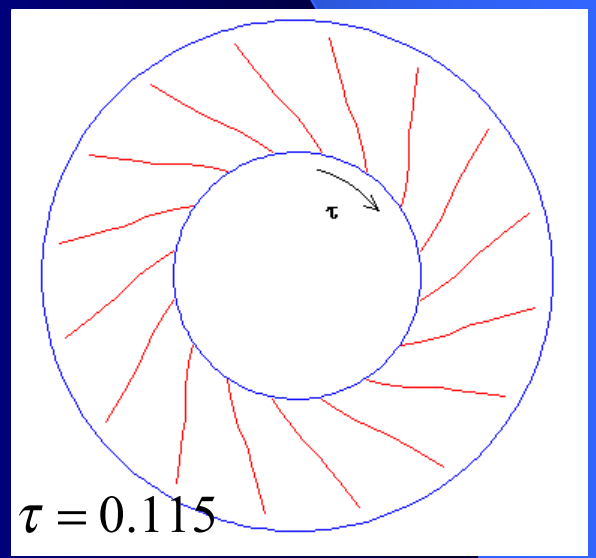


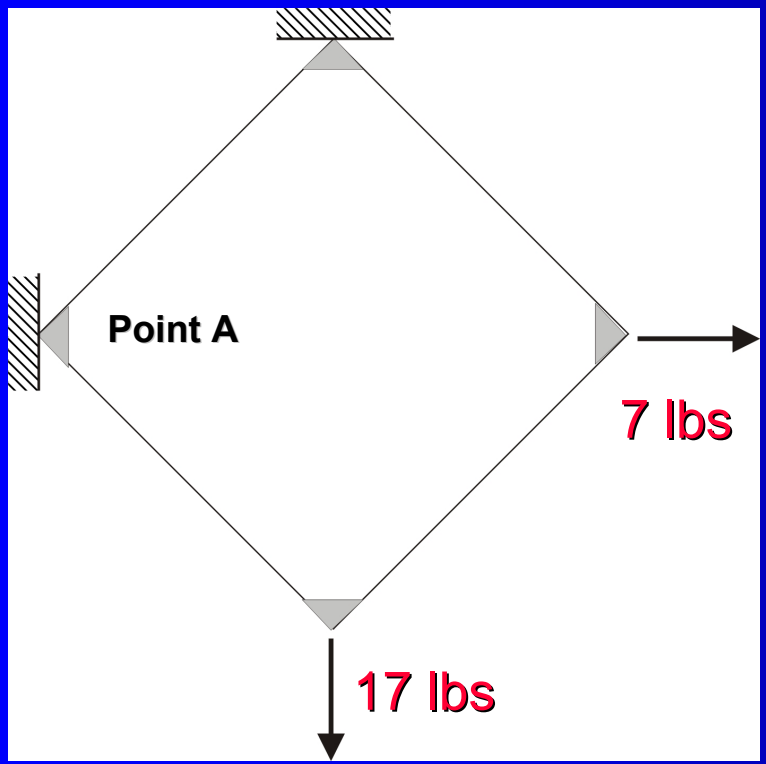



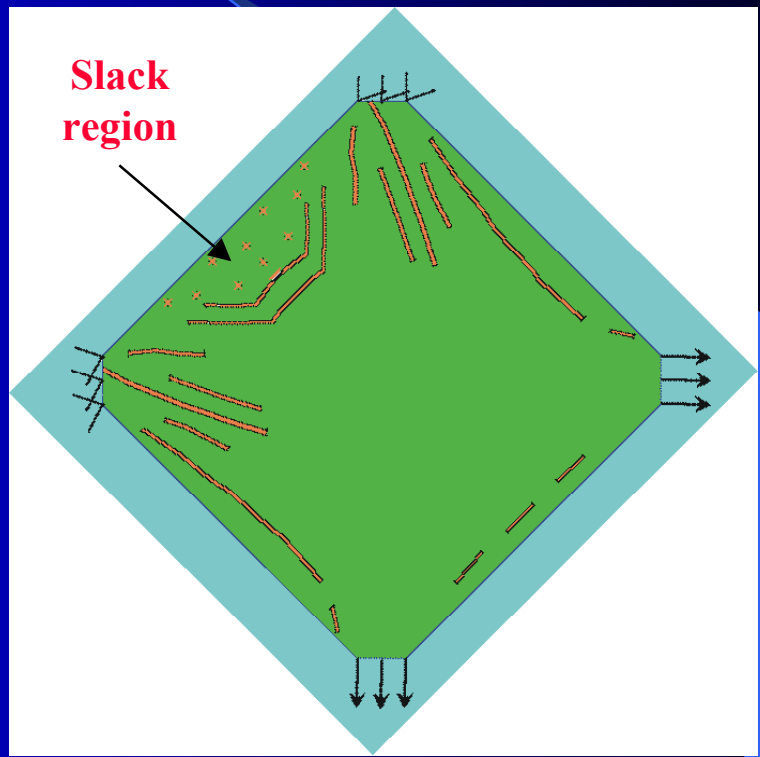



# Preliminary Experiment

- **3 mil aluminized Kapton membrane**
- **800 mm x 800 mm**
- **Loading at corners**

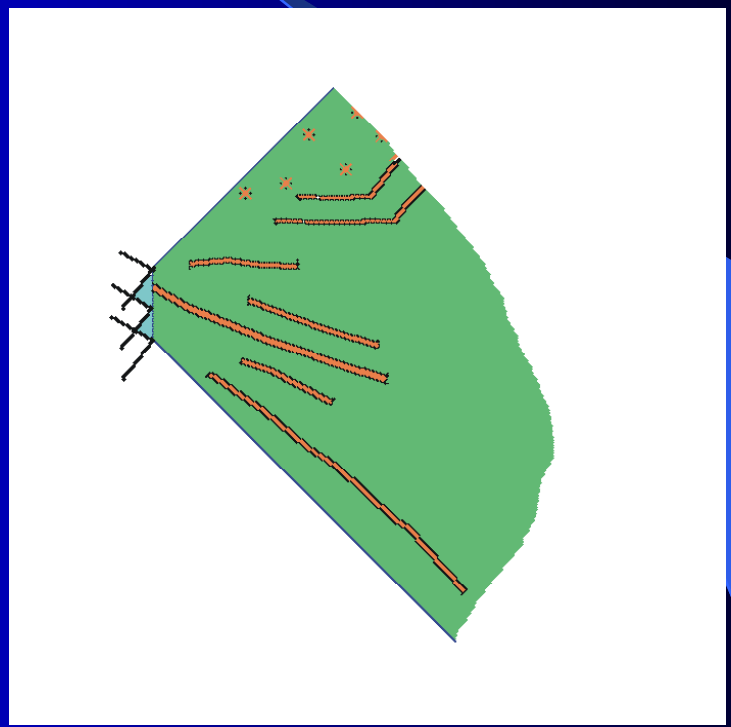
# Simulated Result



# Comparison (at Point A)

Experimental



Numerical

# Conclusions

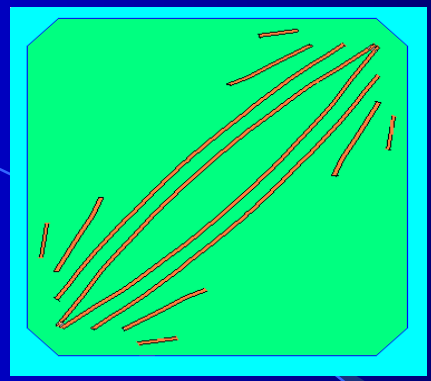
- Wrinkling analysis by FEM with modified elasticity matrix is software-dependent, and needs convergence proof
- The Cable Network Model is good for post-stretching stage ($S_P$) **if** wrinkle patterns are known
- No tools are available to predict out-of-plane deformation and wrinkle waves

# Issues to Resolve In Future Wrinkling Analysis

- Guaranteed convergence
- Complete description of wrinkling process
- Out-of-plane deformation and wrinkle waves

$$\{\sigma\} = [D(\lambda_1, \lambda_2)]\{\varepsilon\}$$

$$\Gamma(\{\lambda\}) \cdot \{\lambda\} = 0$$



## Recommendation:

The two-viable-parameter model with nonlinear math programming scheme is a promising tool for wrinkling analysis and optimal design of gossamer spacecraft


JPL

# THE END